September 29, 2015

7th Grade Module 2 2014-15
Rational Numbers

Topic A: Proportional Relationships

7.NS.A.1

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

d. Apply properties of operations as strategies to add and subtract rational numbers.

Lesson 1: Opposite Quantities Combine to Make Zero

Lesson Helps	Additional Supports/ Materials	I can…
• Integer game can be played as a class instead of in groups. • Laminate (or use sheet protectors) 1-page of number lines (vary blank and numbered) for individual use with white board markers. • Create a number line on the floor using painters tape to model the "counting on" principle. • Provide a wall model of the number line at the front of the room for visual reinforcement.	• Dry erase markers • Number lines (-10 to 10)	• Add positive integers by counting up and negative integers by counting down. • I can understand that the opposite of a number is called the additive inverse because the sum of the two numbers is zero.

Lesson 2-3: Using the Number line to Model the Addition of Integers (can combine these)

Lesson Helps	Additional Supports/ Materials	I can…
• Combine into one lesson using both vertical and horizontal number line to model addition. • Create an anchor poster for the Additive Inverse to help access prior knowledge of number line features including arrow placement and direction and ordering of positive and negative	• Number lines (-10 to 10) • Counters or chips • Number Cards	• Model integer addition on the number line by using arrows to indicate direction of movement. • Recognize that the length of an arrow on the number line is the absolute value of the

SBAC Connection

7.NS.1 – Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

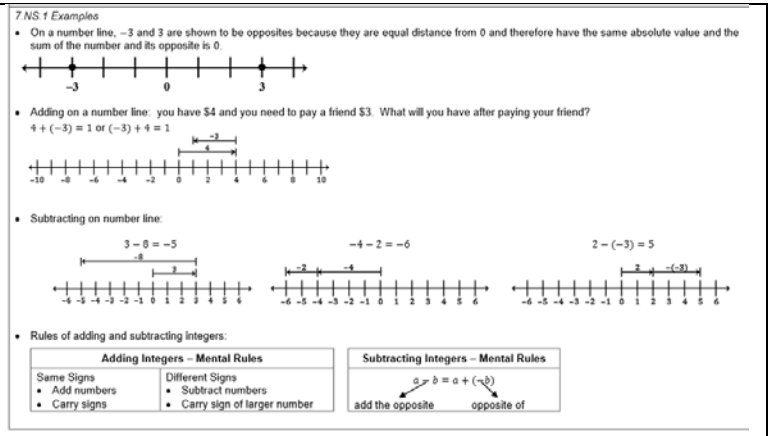
d. Apply properties of operations as strategies to add and subtract rational numbers.

Explanation: This cluster builds upon the understandings of rational numbers in grade 6:

- Quantities can be shown using + or – as having opposite directions or values
- Points on a number line show distance and direction
- Opposite signs of numbers indicate locations on opposite sides of 0 on the number line
- The opposite of an opposite is the number itself
- The absolute value of a rational number is its distance from 0 on the number line
- The absolute value is the magnitude for a positive or negative quantity
- Locating and comparing locations on a coordinate grid by using negative and positive numbers

Students should explore what happens when negatives and positives are combined. Number lines represent a visual image for students to explore and record addition and subtraction results. Two-color counters or colored chips can be used as physical and kinesthetic model for adding and subtracting integers. Repeated opportunities over time will allow students to compare the results of adding and subtracting pairs of numbers, leading to the generalization of the rules. Visual representations may be helpful as students begin this work—they become less necessary as students become more fluent with the operations.

To view or use these examples, copy and paste into a word document.

numbers. • Use counters or chips to transfer prior learning of additive inverse or zero pairs. • Create a number line model on the floor for kinesthetic and visual learners. • Have early finishers explain how absolute value determined the arrow lengths for each of the addends and how they knew each arrow's direction. • Have students use their same cards to create a different addition number sentence and a new number line representation. • Have students examine how the diagram changes when the order of addition changes to reinforce the commutative property. **ELL** • Allow for the use of a number line for ELL students if needed. • Review the concept of "sum" with the whole class for ELL students. • Provide written stems for ELL students. For example, "The sum is ___ units to the ____ of ___."		integer. • Understand addition of integers as putting together or counting up, using the number line. • Use arrows to show the sum of two integers, p + q, on a number line and show that the sum is distance $\|q\|$ from p to the right if q is positive and to the left if q is negative.	

Lesson 4: Efficiently Adding Integers and Other Rational numbers

Lesson Helps	Additional Supports/ Materials	I can…
• Provide some pre-made index cards for learners who struggle forming a question with limited time. • Ask students to refer to anchor posters for support during the game. • Provide pre-made number lines	• Pre-made index cards • Anchor posters (Poster Paper) • Clickers for gauging levels of understanding.	• Understand the rules for adding integers. • Justify the rules using arrows and a number line and extend their findings to begin to include sums of rational

for use throughout the lesson. • Introduce questions one at a time using projection technology to support non-auditory learners. • Use polling software throughout the lesson to gauge the entire class's understanding. • Create anchor posters when introducing integer addition rules. (i.e., Adding Same Sign and Adding Opposite Signs) • Use a gallery wall to post examples and generate student discussion. • To help build confidence, allow students time to "turn and talk" with partners before posing questions.		numbers. • Add integers with the same sign by adding the absolute value and using the common sign. • Add integers with opposite signs by subtracting the smaller absolute value from the larger absolute value and using the sign of the number with the larger absolute value.	
Lesson 5: Understanding Subtraction of Integers and Other Rational Numbers			
Lesson Helps	Additional Supports/ Materials	I can…	
• Display questions and give students time to discuss in their groups prior to whole-class discussion. • Allow students to use whiteboards, number lines, or tables to formulate and justify their opinions to the group. • Record selected student responses and examples on chart paper to help identify patterns. • Allow students to use their Integer Cards throughout this example. • Have students circle the integer with the greater absolute value to determine the final sign of the integer. •	• Whiteboards • Dry erase markers • Number lines • Chart paper • Integer Cards	• Justify the rule for subtraction: Subtraction is the same as adding it's opposite. • Justify the rule for subtraction for all rational numbers from the inverse relationship between addition and subtraction: $(m - n) + n = m$	

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Lesson 6: The Distance Between Two Rational Numbers			
Lesson Helps	Additional Supports/ Materials	I can…	
• Students may find it easier to see the distance if they use a highlighter on the number line and highlight the distance between the two numbers. • **Consider having students determine the distance on the number line first**, and then use the formula to verify (rather than the other way around). • For parts (b)–(e), visual learners will benefit from using the number line to break down the distance into two sections, from zero to each given number.	• Highlighter • Number lines	• Justify the distance formula for rational numbers on a number line (i.e., using p and q to represent variables, $\|p - q\|$). • I know the definition of subtraction in terms of addition and use the definition of subtraction to justify the distance formula. • Solve word problems involving changes in distance or temperature.	
Lesson 7: Addition and Subtraction of Rational Numbers			
Lesson Helps	Additional Supports/ Materials	I can…	
• Number lines without numbers, just line intervals (both horizontal and vertical). • Laminate number lines to use with dry erase markers or visa via pens. • Have students subdivide their number line intervals into 12's and 14's.	• Number lines (paper and laminated) • Dry erase makers	• Recognize that the rules for adding and subtracting integers apply to rational numbers. • Use the number line to model addition, subtraction, and absolute value of integers.	

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Lesson 8 & 9: Applying the Properties of Operations to Add and Subtract Rational Numbers			
Lesson Helps	Additional Supports/ Materials	I can…	
• Select specific cards to give to students to challenge them at their level. • Display an anchor poster in the classroom to show the meaning of "The opposite of a sum is the sum of its opposites." Label the "opposite" and "sum" in a specific math example. • Provide students with a laminate copy of the number line model used in Example 2. Also provide number lines so they can represent each of the following as a sum on the number line.	• Large number line or painter's tape • Poster paper • Laminate copies of number line model • Dry erase markers	• Use properties of operations to add and subtract rational numbers without the use of a calculator • Recognize that any problem involving addition and subtraction of rational numbers can be written as a problem using addition and subtraction of positive numbers only. • Use the commutative and associative properties of addition to rewrite numerical expressions in different forms.	

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Topic B: Multiplication and Division of Integers and Rational Numbers

7.NS.A.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal number using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Lesson 10: Understanding Multiplication of Integers

Lesson Helps	Additional Supports/ Materials	I can…
• Students practice and justify their understanding of multiplication of integers by using the Integer Game. • Integer game can be played as a class.	• Integer game revisited (Integers Cards)	• understand the rules for multiplication of integers and that multiplying the absolute values of integers result in the absolute value of the product. • understand that (-1)(-1) = (1), and see that it can be proven to be true mathematically through the use of distributive property and the additive inverse.

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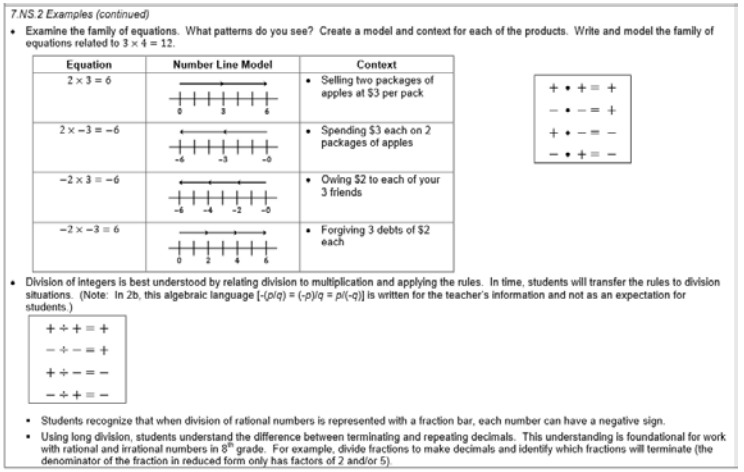


7.NS.2 Examples (continued)

- Examine the family of equations. What patterns do you see? Create a model and context for each of the products. Write and model the family of equations related to $3 \times 4 = 12$.

Equation	Number Line Model	Context
$2 \times 3 = 6$		• Selling two packages of apples at $3 per pack
$2 \times -3 = -6$		• Spending $3 each on 2 packages of apples
$-2 \times 3 = -6$		• Owing $2 to each of your 3 friends
$-2 \times -3 = 6$		• Forgiving 3 debts of $2 each

$+ \cdot + = +$
$- \cdot - = +$
$+ \cdot - = -$
$- \cdot + = -$

- Division of integers is best understood by relating division to multiplication and applying the rules. In time, students will transfer the rules to division situations. (Note: In 2b, this algebraic language $[-(p/q) = (-p)/q = p/(-q)]$ is written for the teacher's information and not as an expectation for students.)

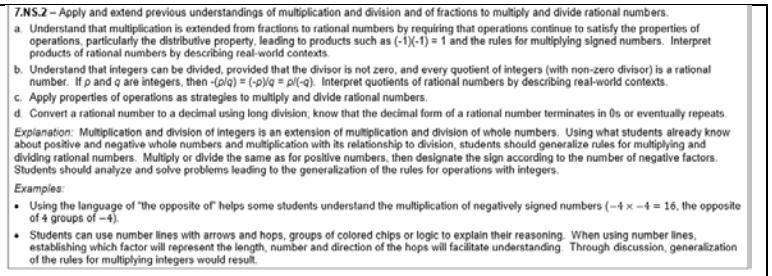
$+ \div + = +$
$- \div - = +$
$+ \div - = -$
$- \div + = -$

- Students recognize that when division of rational numbers is represented with a fraction bar, each number can have a negative sign.
- Using long division, students understand the difference between terminating and repeating decimals. This understanding is foundational for work with rational and irrational numbers in 8th grade. For example, divide fractions to make decimals and identify which fractions will terminate (the denominator of the fraction in reduced form only has factors of 2 and/or 5).

Lesson 11: Develop Rules for Multiplying Signed Numbers

Lesson Helps	Additional Supports/ Materials	I can…
• Students describe, using Integer Game scenarios • Create an "anchor poster" showing the quadrants with the new rules for multiplying integers. • Use color or highlight steps to help students organize and understand the manipulations. • Create teacher/student T-chart	• Chart paper	• Use the rules for multiplication of signed numbers and give real-world examples. • Use the constant of proportionality to represent proportional relationships by equations in real world

To view or use these examples, copy and paste into a word document.

on which the teacher writes a real -world situation that corresponds with a product, and students write similar situations using different numbers.		extents as they relate the equations to a corresponding ratio table and/or graphical representation.	
Lesson 12: Division of Integers			
Lesson Helps	Additional Supports/ Materials	I can…	
• Integer multiplication facts bubble. • Fact fluency can be done more than once so students can see their growth.		• Recognize that division is the reverse process of multiplication. That integers can be divided provided the divisor is not zero. If p and q are integers, than –(p/q) = -p/q = p/-q.	
Lesson 13 - 14: Converting Between Fractions & Decimals (Long Division)			
Lesson Helps	Additional Supports/ Materials	I can…	
• Combine these two lessons. • Provide or create a place value chart to aid those who do not remember their place values or for **ELL students** who are unfamiliar with the vocabulary. • Have students create a graphic organizer to relate the different representations of rational numbers including fraction, decimals, and words. Pictures may also be used if applicable. • Terminating" and "non-terminating."	• Graph Paper	• Understand that the context of a real-life situation often determines whether a rational number should be represented as a fractional or decimal. • Understand that decimals specify points on the number line by repeatedly subdividing intervals into tenths. • Convert positive	

7.NS.2 – Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Explanation: Multiplication and division of integers is an extension of multiplication and division of whole numbers. Using what students already know about positive and negative whole numbers and multiplication with its relationship to division, students should generalize rules for multiplying and dividing rational numbers. Multiply or divide the same as for positive numbers, then designate the sign according to the number of negative factors. Students should analyze and solve problems leading to the generalization of the rules for operations with integers.

Examples:

- Using the language of "the opposite of" helps some students understand the multiplication of negatively signed numbers ($-4 \times -4 = 16$, the opposite of 4 groups of -4).
- Students can use number lines with arrows and hops, groups of colored chips or logic to explain their reasoning. When using number lines, establishing which factor will represent the length, number and direction of the hops will facilitate understanding. Through discussion, generalization of the rules for multiplying integers would result.

• Review vocabulary of long division, i.e., algorithm, dividend, divisor, remainder. • For long division calculations, provide students with graph paper to aid their organization of numbers and decimal placement.		decimals to fractions and fractions to decimals when the denominator is a product of only factors of 2 and/or 5. • Understand that every rational number can be converted to a decimal. • Represent fractions as a decimal numbers that either terminate in zeros or repeat, representing repeating decimals using a bar over the shortest sequence of repeating digits. • Interpret word problems and convert between fraction and decimal forms or rational numbers.	
Lesson 15 - 16: Multiplication and Division of Rational Numbers			
Lesson Helps	Additional Supports/ Materials	I can…	
• Combine these two lessons. • For students who are not yet fluent with integer multiplication, provide cards with the rules for integer multiplication. • Remind students that "the opposite of a sum is equivalent to the sum of its opposites."	• White boards • Index cards • Dry erase markers	• Recognize that the rules for multiplying and dividing integers apply to rational numbers. • Interpret products and quotients of rational numbers by describing real-world contexts. • Use the properties of operations	

		(commutative, associative, and distributive properties) to multiply and divide rational numbers without the use of a calculator.	

Mid-Module Assessment and rubric can be used at your discretion.

Topic C: Applying Operations with Rational Numbers to Expressions and Equations

7.NS.A.3, 7.EE.A.2, 7.EE.B.4a

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05".

7.EE.B.4a Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about quantities.

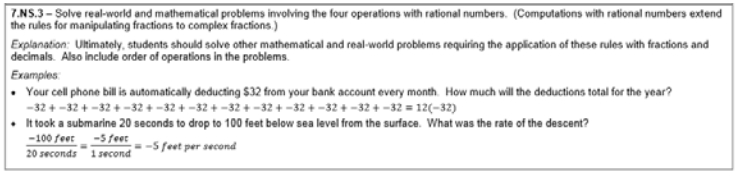
a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Lessons 17: Comparing Tape Diagram Solutions to Algebraic Solutions

SBAC Connections

Lesson Helps	Additional Supports/ Materials	I can…
• Review how to set up a tape diagram when given the parts and total. • Scaffolding: Review from 6th grade solving 1-step and 2-step equations algebraically as well as the application of the distributive property.	• Tape Diagram (pre-made if needed)	• Use tape diagrams to solve equations of the form px + q = r and p(x + q) = r, and identify the sequence of operations used to find the solution. • Translate word problems to write and solve algebraic equations using tape diagrams to model the steps they record algebraically.

To view or use these examples, copy and paste into a word document.



7.NS.3 – Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

Explanation: Ultimately, students should solve other mathematical and real-world problems requiring the application of these rules with fractions and decimals. Also include order of operations in the problems.

Examples:

- Your cell phone bill is automatically deducting $32 from your bank account every month. How much will the deductions total for the year?
  $-32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 = 12(-32)$
- It took a submarine 20 seconds to drop to 100 feet below sea level from the surface. What was the rate of the descent?
  $\frac{-100\ feet}{20\ seconds} = \frac{-5\ feet}{1\ second} = -5\ feet\ per\ second$

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Lessons 18 - 19: Writing, Evaluating, and Finding Equivalent Expressions with Rational Numbers

Lesson Helps	Additional Supports/ Materials	I can…
• Tic-Tac-Toe Review • Tape diagram for representing percent as a fraction of a whole.	• Tic-Tac-Toe grids (pre-made or students can create) • Tape diagrams	• Create equivalent forms of expressions in order to see structure, reveal characteristics, and make connections to context. • Compare equivalent forms of expressions and recognize that there are multiple ways to represent the context of a word problem. • Write and evaluate expressions to represent real-world scenarios.

Lesson 20: Investments–Performing Operations with Rational Numbers

Lesson Helps	Additional Supports/ Materials	I can…
• Scaffolding: Discuss what a register is, how it is used to organize a series of transactions. Also, discuss how a loss can be represented by using parenthesis (e.g., (607.29)). • Scaffolding: Review or reiterate that the operation associated with payments is subtraction, and the operation associated with deposits is addition.		• Perform various calculations involving rational numbers to solve a problem related to the change in an investment's balance over time. • Recognize and use mathematics as a tool to solve real-life problems.

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Lesson 21: If-Then Moves with Integer Number Cards			
Lesson Helps	Additional Supports/ Materials	I can…	
•	• Integer Cards		
Lessons 22–23: Solving Equations Using Algebra			
Lesson Helps	Additional Supports/ Materials	I can…	
• Scaffolding: Have students write out in words what they will do to help them transition from words to algebraic symbols. • Scaffolding: Provide a review card showing examples of fraction multiplication and division for students who do not have adequate prerequisite skills.	• Tape diagrams are optional		
End of Module Assessment			